Initial Relative Orbit Determination of Space Objects via Radio Frequency Signal Localization

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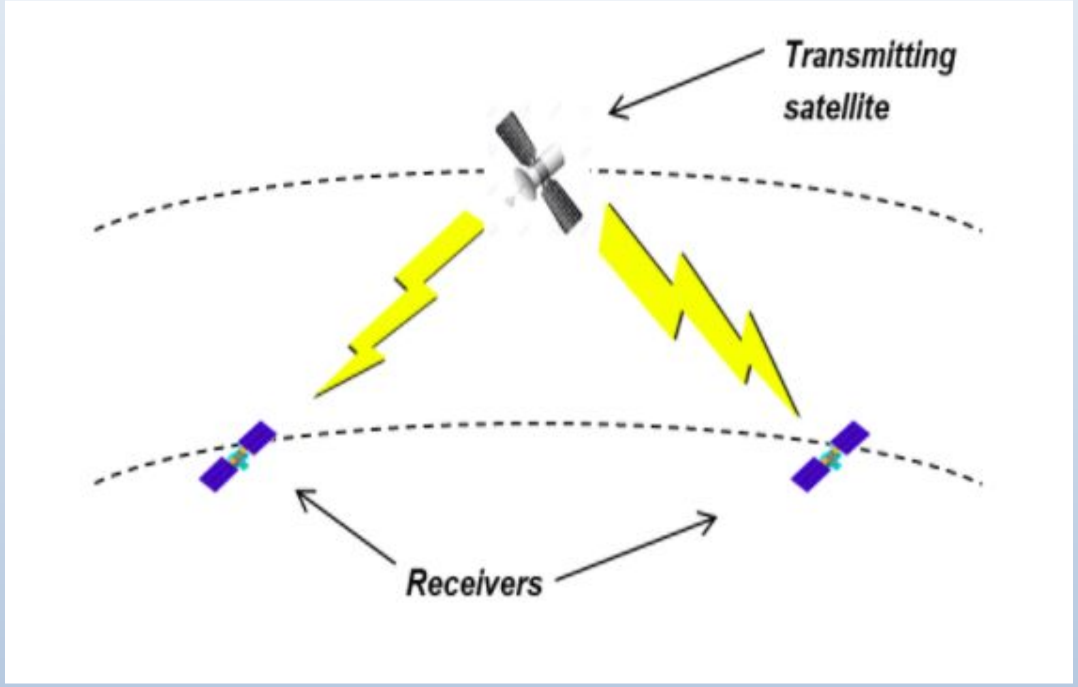
Background

- **RESEARCH GOAL**: To improve the precise localization of electromagnetic transmitting sources in Earth-orbiting space, from receivers that are also in orbit
- The focus of this work could best be described as passive or uncooperative localization, whereby there is no coordination between the transmitter(s) and receiver(s)
- Localization is performed based only on knowledge of the received signal itself
  - e.g. amplitude, phase, and frequency
- A time difference of arrival (TDOA) measurement is used to relate the transmitter location to the receiver locations
  - TDOA multiplied by signal travel speed (speed of light) yields range difference of arrival (RDOA)
  - A polynomial system can then be developed and solved using various root-finding methods

Space-to-Space Problem

- Localization problem involving only 2 receivers
  - Can be performed with more than 2
- Both the transmitter & receivers are in Earth orbit
  - Different from geolocation
- For a space-to-space scenario, one of the receivers is chosen as a “reference” satellite
  - Each RDOA equation is written in this receiver’s relative (LVLH) frame
- The solution of the localization problem is determining the initial relative orbit of the transmitter



Polynomial Derivation

- Previous work shows that the TDOA equation is a second order polynomial in terms of the transmitter's instantaneous location coordinates $x(t), y(t), z(t)$
- The Clohessy-Wiltshire solution is a closed-form expression of the transmitter's relative orbit w.r.t. the reference orbit
  - Dependent on transmitter's relative orbit at initial (epoch) time, defined as $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$
- Inserting the $x(t), y(t), z(t)$ CW solution into the TDOA equation transforms it into a second order polynomial in terms of these 6 parameters
- Provides a way to analytically check that coefficients produced are accurate

Scenario Setup

- For a space-to-space scenario, one of the receivers is chosen as a "reference" satellite, & each RDOA equation is written in this receiver's relative (LVLH) frame
- Our problem is then one of initial relative orbit determination (IROD)
- If we choose Receiver 1 as the reference, then the inputs/knowns are:
  - Receiver 2 location (relative to Receiver 1) at each measurement time
  - Range difference of arrival (RDOA) values at each measurement time
  - Note that Receiver 1 location (relative to Receiver 1) at each measurement time is ZERO

Scenario Setup

- For this work, we chose to focus on planar (2D) scenarios
  - Transmitter and both receivers are all coplanar, therefore $z_0 = \dot{z}_0 = 0$
  - Unknowns are $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0$
  - 3D cases have been solved, but not in time for publication here
- A system of 4 second order polynomials is derived with 15 terms in each polynomial
  - ${x_0}^2, x_0 y_0, x_0 \dot{x}_0$, etc
  - 4 measurement times are required to solve for the 4 unknowns
- Two scenarios are presented

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Solving Scenarios

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- Bezout's Theorem states that there are 16 (finite) solutions to the polynomials
  - Technically it states that there are (highest degree of the polynomial)*(number of polynomial equations) finite solutions, but in our planar case 2^4=16
- To solve these coupled multivariate polynomials, we employ an early 20th-century linear algebra-based technique developed by Macaulay (implemented in MATLAB), and a numerical homotopy continuation technique called Bertini
- Polynomial scaling is included to improve solution accuracy

Solving Scenarios: Macaulay's Method

- Re-write the system of equations in terms of an anchor variable
  - Typically $\dot{y}_0$ in the planar case or $\dot{z}_0$ in the full 3D case, treated as a constant
- A homogenization step makes the polynomial degree equal by adding a variable, such that the coefficients and unknown monomials becomes

$$\left(\mathbf{M}_2\dot{y}^2 + \mathbf{M}_1\dot{y} + \mathbf{M}_0\right)\chi = 0$$

- Which can be solved as the generalized eigenvalue problem where the real, non-infinite eigenvalues are possible solutions for the anchor variable

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_0 & -\mathbf{M}_1 \end{bmatrix}\nu = \dot{y}\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 \end{bmatrix}\nu$$

- There are 112 eigenvalues output, with an expected 16 finite values (though this is not always the case if the process lacks the necessary precision)

Solving Scenarios: Bertini

- Whereas Macaulay is an analytical (non-iterative) method, Bertini is a numerical (iterative) method
- Begins with a number of “paths” equal to the Bezout number (in our case 16)
- Each path undergoes an intricate process of convergence to (hopefully) one of the solutions
- Convergence not guaranteed, a path may diverge to infinity
- Typically Bertini solutions tend to be more accurate than Macaulay, & do not seem to require scaling of the polynomials

Solving Scenarios: Polynomial Scaling

- Experience has demonstrated that the system of equations are poorly conditioned polynomials, requiring high precision for Macaulay to accurately solve
  - High condition number, 10^9
  - The magnitudes of the coefficients (from smallest to largest) encompass several orders of magnitude
- Similar to Morgan's SCLGEN algorithm, we scale the polynomials
  - Variable substitution (scaling)
  - Equation scaling
  - Center coefficients around unity while simultaneously minimizing the variance
  - Sum of squares means a global minimum can be found analytically

$$10^{c_N}\left[10^{2c_1}a_1\bar{x}_T^2 + 10^{c_1}10^{c_2}a_2\bar{x}_T\bar{y}_t + \ldots a_{28}\right] = 0$$

Solving Scenarios: Ordering

Method 1: Macaulay on original (unscaled) system

Method 2: Macaulay on scaled system

Method 3: Bertini on original (unscaled) system

Method 4: Bertini on scaled system

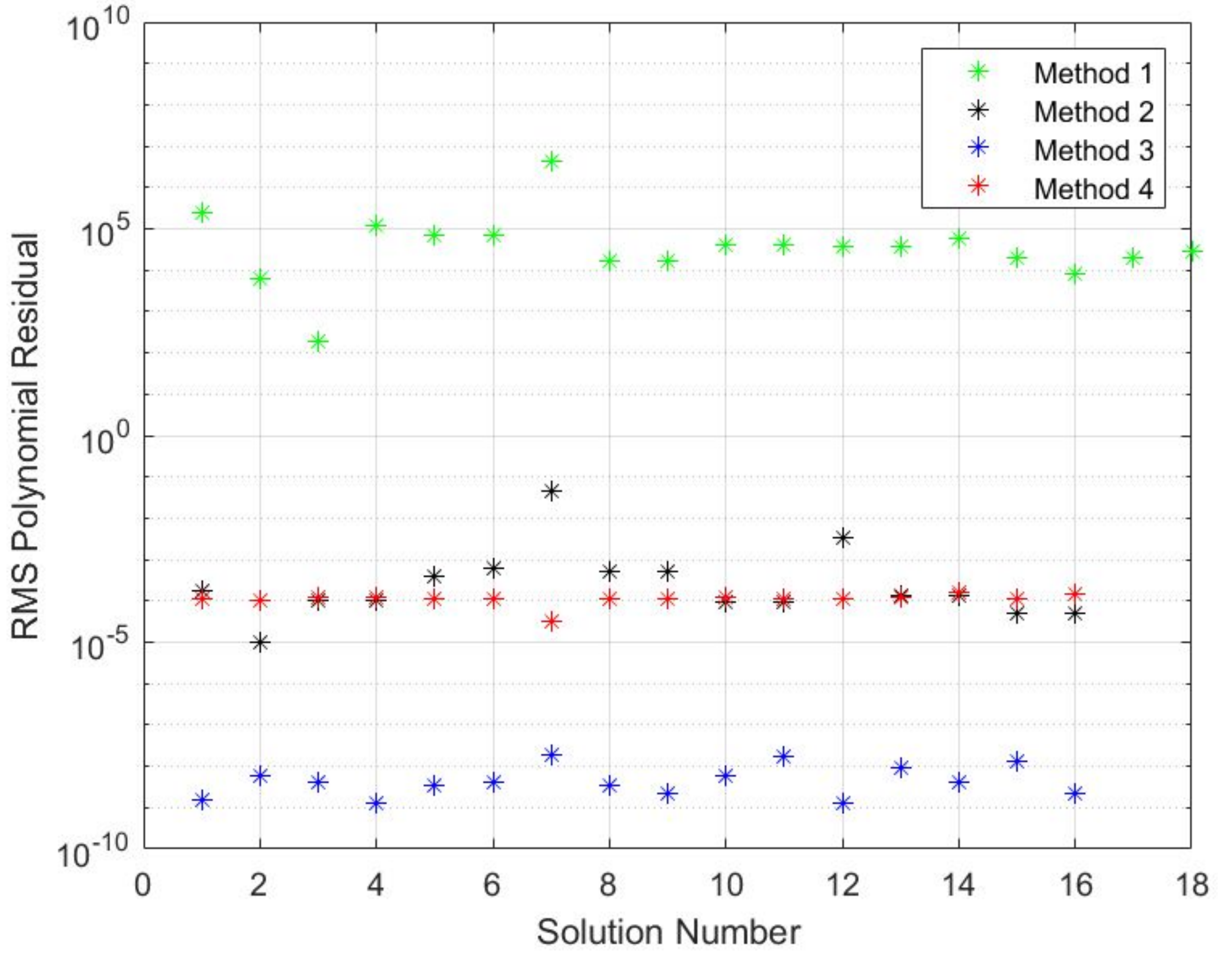
Scenario 1

- Consider the initial conditions

	Transmitter State (1)	Receiver 1 State	Receiver 2 State
$x(t_0), km$	1	10	8
$y(t_0), km$	11	-5	3
$\dot{x}(t_0), km/s$	0.012	0.001	0.01
$\dot{y}(t_0), km/s$	0.03	-0.02263	-0.008102

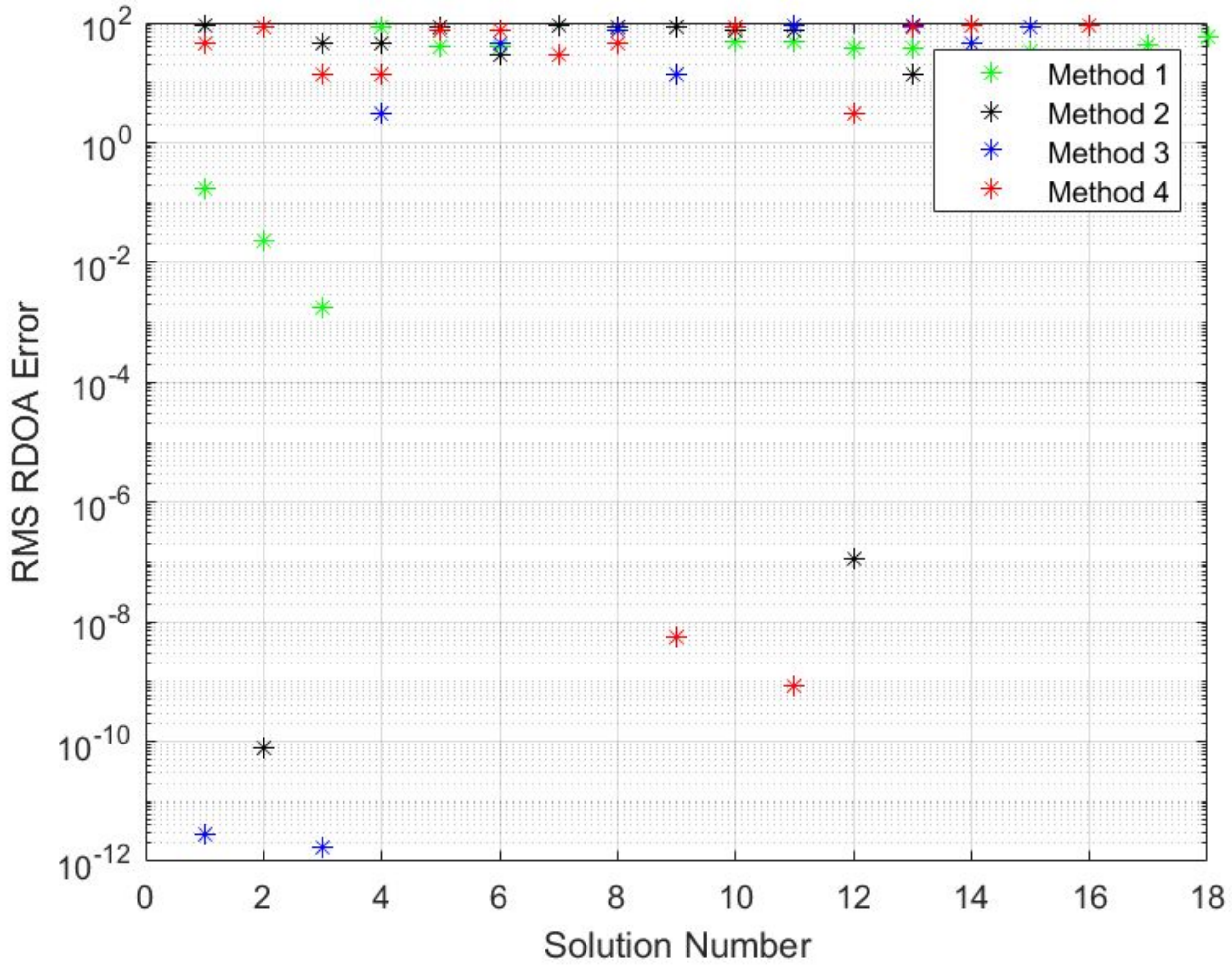
Scenario 1

- Polynomial Residuals



Scenario 1

- RDOA Residuals



Scenario 1

- Minimum RDOA solutions

Value	Method 3	Method 4	Method 1
$x(t_0)_1, km$	0.9999+2.2039e-15i	1.0000+1.8415e-16i	1.0000
$y(t_0)_1, km$	11.0000-3.1296e-15i	11.0000+2.9633e-16i	11.0000
$\dot{x}(t_0)_1, km/s$	0.0120-1.4352e-18i	0.0120+3.1000e-19i	0.0120
$\dot{y}(t_0)_1, km/s$	0.0300-1.2766e-17i	0.0300+3.0262e-19i	0.0300
$x(t_0)_2, km$	4.4420+4.1027e-19i	4.4420-5.7957e-14i	4.4420
$y(t_0)_2, km$	6.2422-1.0081e-18i	6.2422+7.7120e-14i	6.2422
$\dot{x}(t_0)_2, km/s$	0.0247-2.3899e-21i	0.0247+2.4027e-16i	0.0247
$\dot{y}(t_0)_2, km/s$	-0.0009-1.1907e-21i	-0.0009+1.5554e-16i	-0.0009

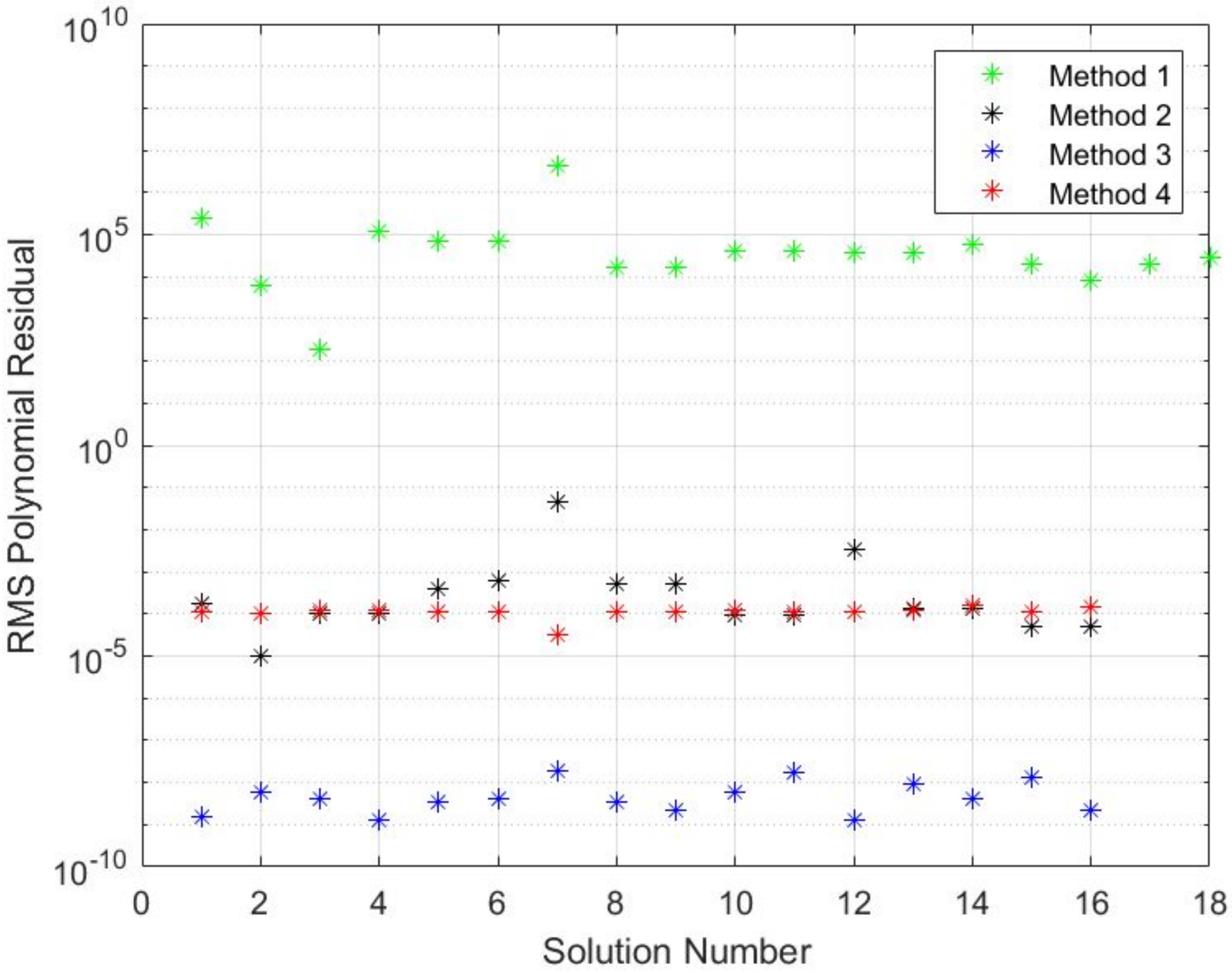
Scenario 2

- Consider the initial conditions

	Transmitter State (2)	Receiver 1 State	Receiver 2 State
$x(t_0), km$	4.5	10	8
$y(t_0), km$	7	-5	3
$\dot{x}(t_0), km/s$	-0.022	0.001	0.01
$\dot{y}(t_0), km/s$	0.05	-0.02263	-0.008102

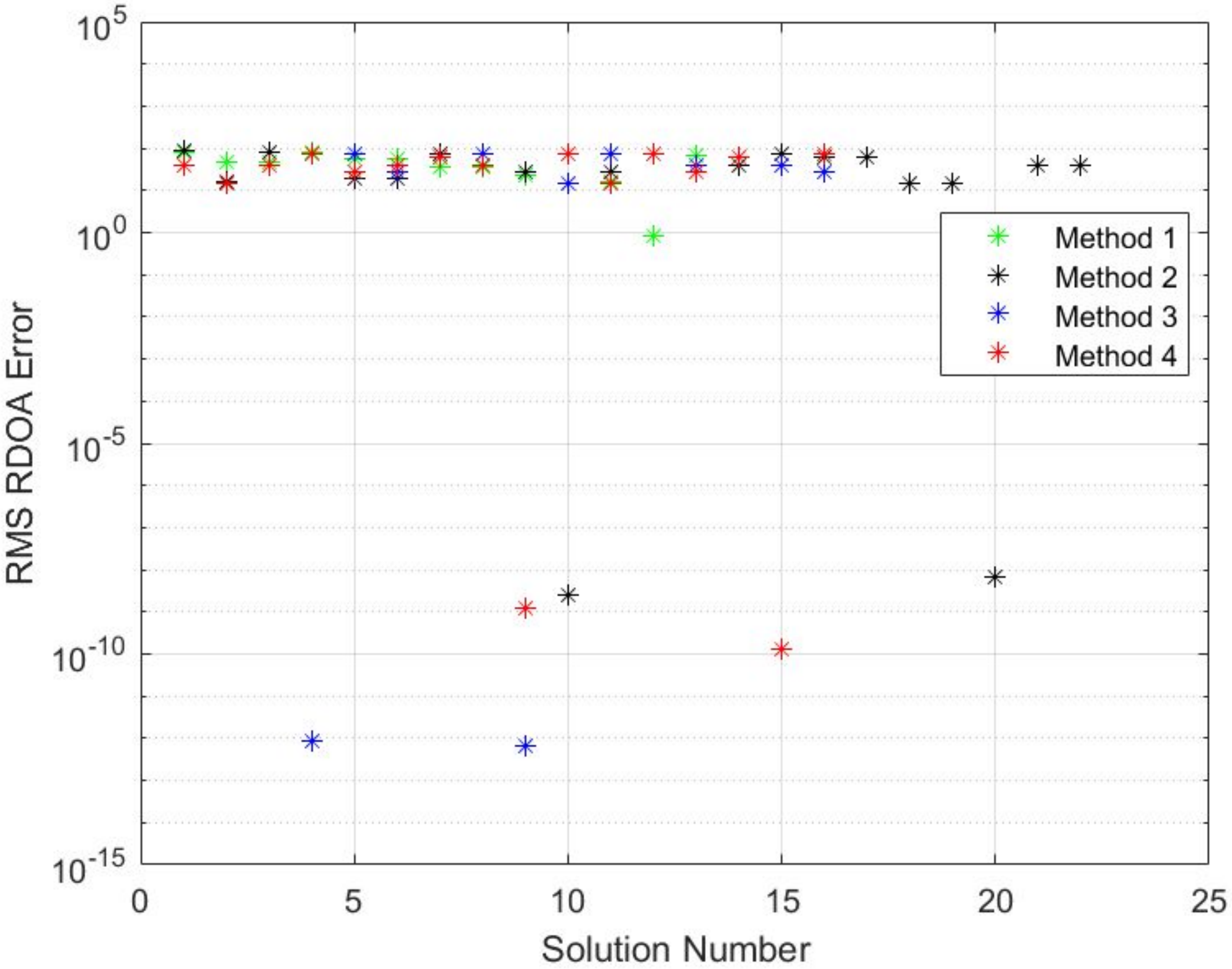
Scenario 2

- Polynomial Residuals



Scenario 2

- RDOA Residuals



Scenario 2

- Minimum RDOA solutions

Value	Bertini	Bertini Scaled	Macaulay Scaled
$x(t_0)_1, km$	4.5000-1.3446e-15i	4.5000+2.3592e-15i	4.5000
$y(t_0)_1, km$	7.0000+3.8100e-15i	7.0000-3.0616e-15i	7.0000
$\dot{x}(t_0)_1, km/s$	-0.02200-1.7369e-18i	-0.0220+2.2918e-18i	-0.0220
$\dot{y}(t_0)_1, km/s$	0.0500+1.0680e-17i	0.0500-7.4323e-18i	0.0500
$x(t_0)_2, km$	-7.3008-6.3417e-17i	-7.3008-2.0800-14i	-7.3008
$y(t_0)_2, km$	26.2213+1.0809e-16i	26.2213+3.5745e-14i	26.2213
$\dot{x}(t_0)_2, km/s$	0.08202+3.0028e-19i	0.08211+9.4893e-17i	0.0820
$\dot{y}(t_0)_2, km/s$	0.003815+7.0990e-20i	0.003815+2.4885e-17i	0.0038

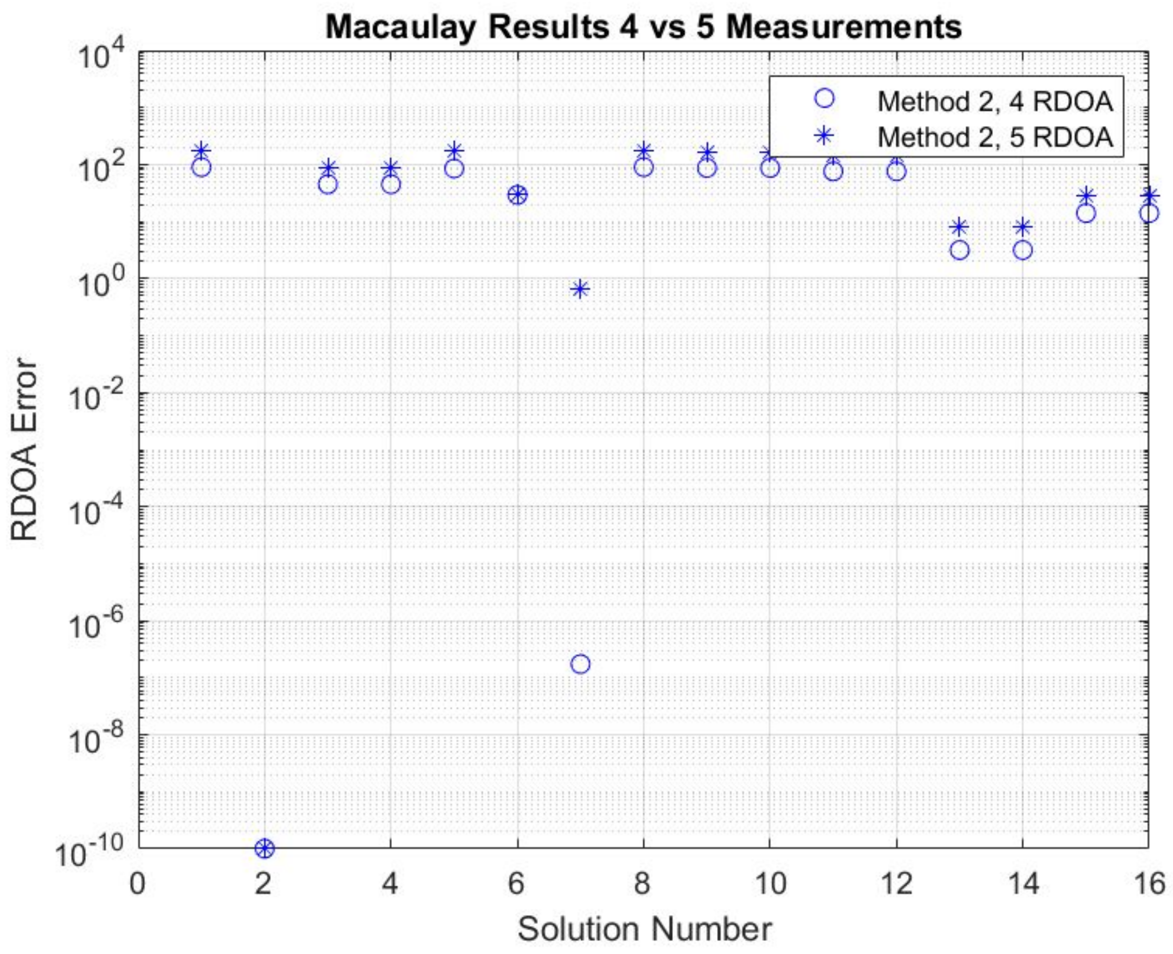
Solution Disambiguation

- Methods 2, 3, and 4 produced two results each that resulted in very low RDOA residuals
- The downside is that both results appeared feasible

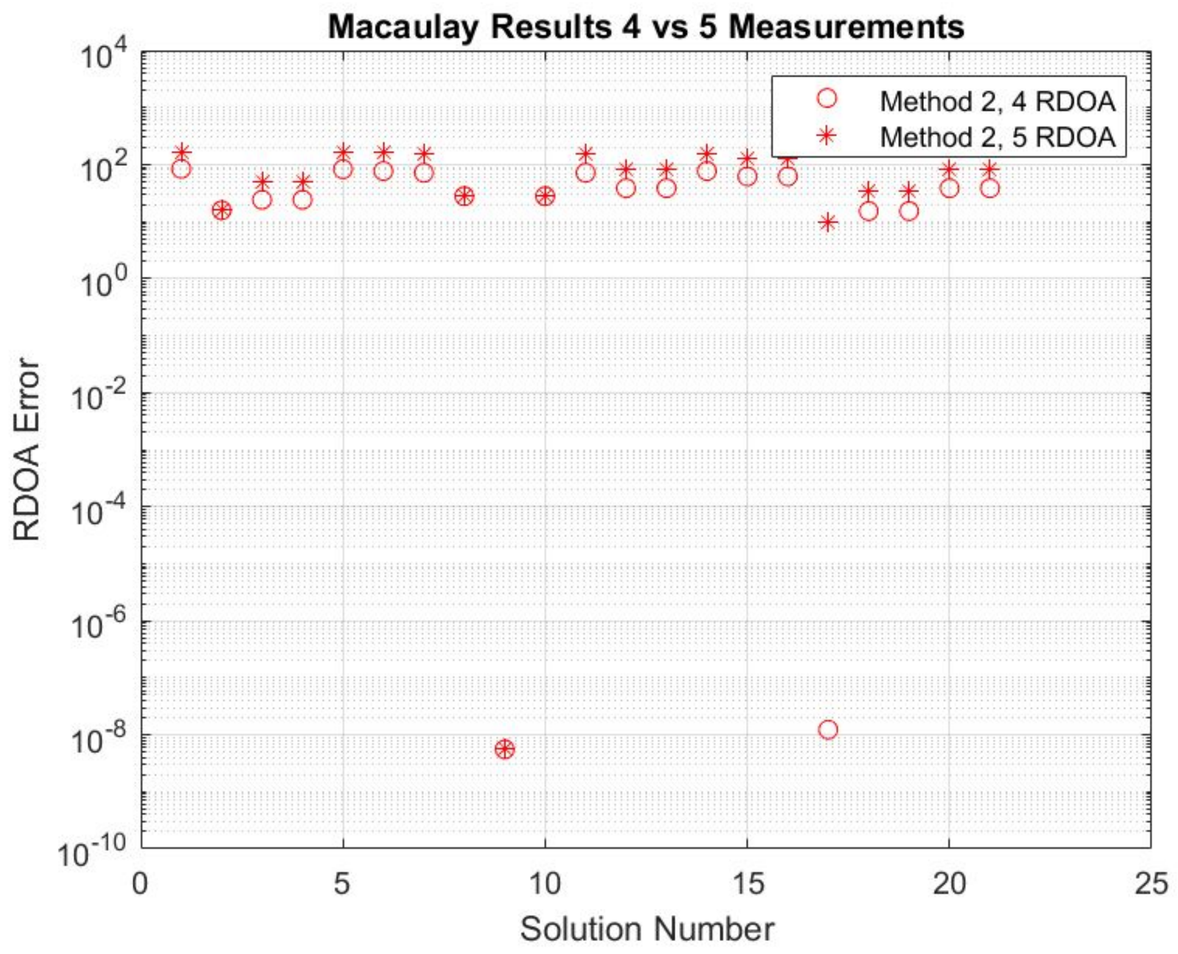
- By adding a 5th measurement, we should be able to distinguish which is the true solution because only it should follow the dynamics
- Result is that the true solution is determined to within 10^{-9} meters and 10^{-8} m/s and the additional solution may be discarded

Solution Disambiguation

Scenario 1



Scenario 2



Conclusions and Future Work

- Astrolocation via RF transmission has been defined, measurement converted to a polynomial system, and solutions provided via multiple methods
- Continued research will include:
  - 3D scenarios (unscaled and scaled)
  - Adjusting solution precision (in MATLAB or other language)
  - Computation time and implementation into flight-like hardware
  - Statistical uncertainty models